Probabilistic Structural Analysis to Quantify Uncertainties Associated with Turbopump Blades

Vinod K. Nagpal* and Robert Rubinstein†
Sverdrup Technology, Inc., Middleburg Heights, Ohio
and
Christos C. Chamis†
NASA Lewis Research Center, Cleveland, Ohio

A probabilistic study of turbopump blades has been in progress at NASA Lewis Research Center for over the last three years. The objective of this study is to evaluate the effects of uncertainties in geometry and material properties on the structural response of the turbopump blades. The eventual purpose is to evaluate the tolerance limits on the design. A probabilistic method has been developed to quantify the effects of the random uncertainties. The results of this study indicate that only the uncertainties in geometry have significant effects on the structural response from among the variables included in the analyses.

Introduction

GEOMETRY and material properties are generally considered deterministic when designing or analyzing structural components. In fact, geometry, material properties, and loading are uncertain. These uncertainties, random in nature, can be considered as variables, which are often conservatively accounted for in the design by a factor of safety. It is true that these uncertainties are small in magnitude, but they sometimes have a significant effect on the performance of critical components such as turbine blades. Therefore, it is very important to quantify their influence on the structural response.

The initial purpose of this study is to develop a methodology to quantify the influence of these random variations on the performance of structural components (uncertainties). A second purpose is to apply this methodology to quantify uncertainties in geometry, material properties, and loadings. The component selected for the study is Space Shuttle Main Engine (SSME) turbopump blade (Fig. 1). As a first phase of the study, variations associated with geometry and material properties have been investigated. Quantifying the influence of variations in the loadings is underway and will be reported in the near future.

Many studies have demonstrated that probabilistic analysis methods for components under random loading are more reliable than deterministic approaches. Probabilistic methods have been predominantly used in fatigue, fracture mechanics, and structural reliability analyses under random vibrations.[1-5] Particularly in fatigue, improvements in estimating fatigue life ranged up to several hundred percent in comparison to a widely used deterministic approach.[6] Because of the potential of probabilistic methods, their application has been accepted in other fields of engineering in recent years. These fields include input loading, finite elements, and metallurgy. A probabilistic approach to develop a composite loading for SSME components is underway at the Battelle Laboratories,[7] and has been discussed by Shinozuka and Tan[8] and Lin.[9] Other studies using probabilistic finite-element methods have been briefly reviewed by DasGupta.[10] A variational approach for developing the probabilistic finite elements is under development by Belytschko.[11] The usefulness and importance of the probabilistic approach, especially for turbopump blades, has been summarized by Chamis.[12]

The influence of the variations associated with geometry and material properties in this study were quantified by simulation. All the experimental runs were simulated on a computer. No real experiments were performed. Conducting experiments for such studies is extremely complex and expensive, and in some cases, impossible even with start-of-the-art methods.

Analysis Procedure

The influences of random variations in geometry and material properties have been studied by modifying and using a NASA-Lewis finite-element code known as STAEBL (Structural Tailoring of Engine Blades). STAEBL modeled the airfoil of the turbopump blade with 80 triangular shell elements consisting of 55 nodes. The blade dimensions are $1.25 \times 1.00 \times 0.25$ in. A typical blade geometry is shown in Fig. 1. The Monte Carlo simulation technique was used in this study.

The influences of random variations, both in geometry and material properties, were quantified in terms of variations in structural response of the blade. The structural response of the blade was characterized in terms of three variables: the first three natural frequencies, stress at the fixed end (root stress), and lateral blade tip displacements. The first three natural frequencies were considered sufficient for the methodology development.

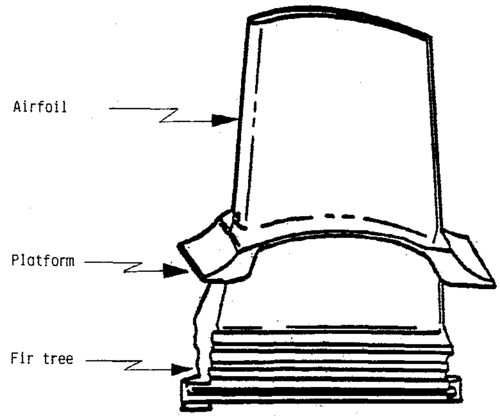

Fig. 1 Typical turbine blade geometry.

Presented as Paper 87-0766 at the AIAA/ASME/AHS/ASEE 28th Structures, Structural Dynamics, and Materials Conference, Monterey, CA, April 6-8, 1987; received July 23, 1987; revision received July 1, 1988.

*Department Director.

†Section Supervisor.

‡Senior Research Scientist. Associate Fellow AIAA.

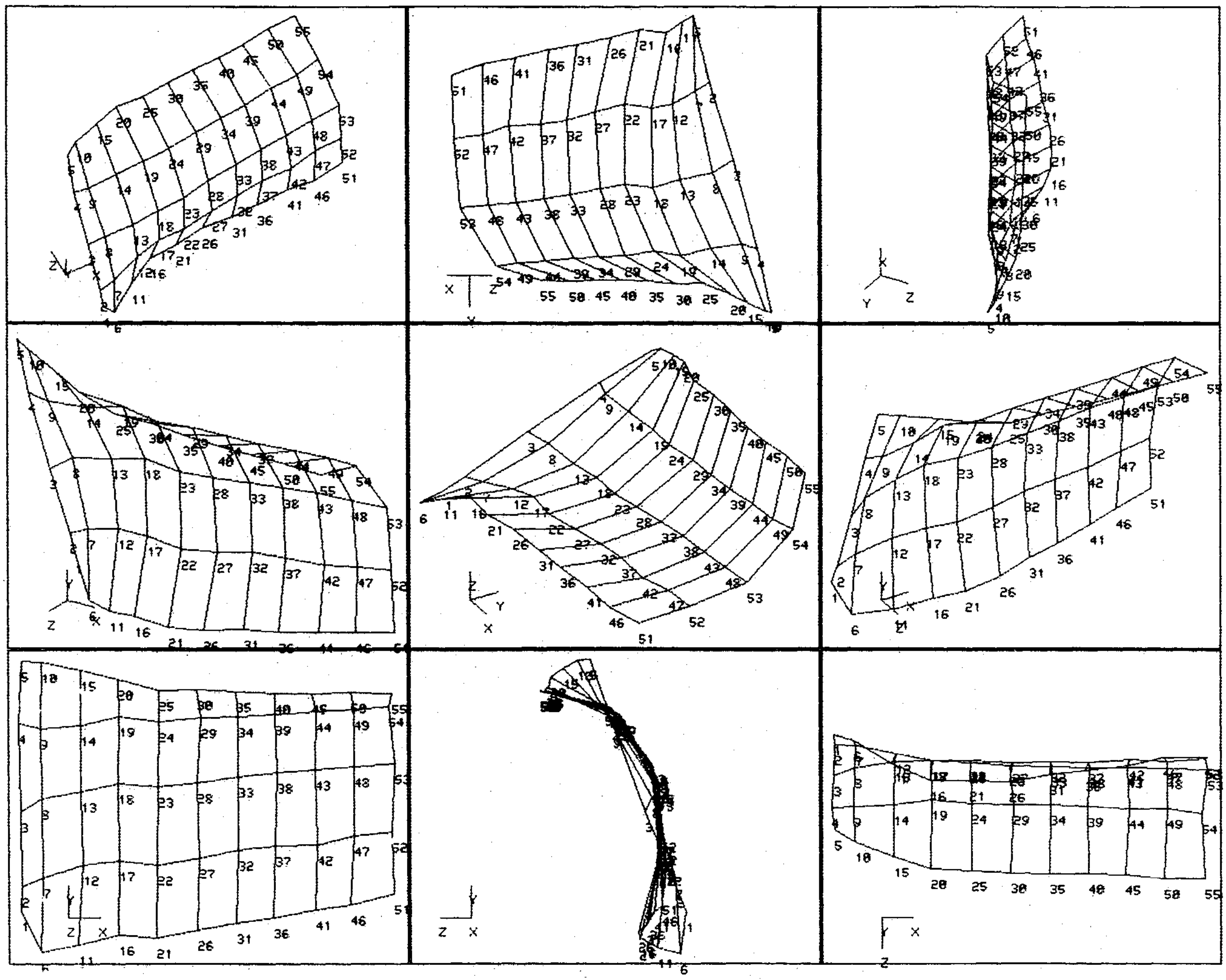

Fig. 2 Perturbed geometry (exaggerated).

Geometry uncertainties were simulated by randomly perturbing the nodes of the finite-element mesh. Geometrical coordinates of each node were perturbed with the help of random numbers generated from a computer. The maximum perturbation magnitude was limited to 10% of the original magnitude. This perturbation was performed in all three (x, y, and z) directions. A sample plot of the perturbed geometry is shown in Fig. 2.

Material-property uncertainties were simulated by perturbing the elements of the material-property matrix for each finite element. Since the blade was very thin and the property matrix is symmetrical, only six components, which are related to in-plane directions, were perturbed. The maximum perturbation was once again limited to 10% of the original values. The maximum perturbation of 10% was justified by previous experience and experiments. In perturbing the properties, normally distributed uncorrelated random numbers were used.

To determine the influence of interactions, geometry and material properties were perturbed simultaneously. The reason this study was performed in three simple steps was to eliminate the insignificant variables which otherwise would have made the study unnecessarily complex.

This is the first part of the three-part study. In this part, only uncorrelated randomness in geometry and material properties have been studied. Random variation in these two variables can be uncorrelated or correlated depending upon the manufacturing process, machines, and working personnel. The second part of this study will include correlated random variation. The third part will deal with randomness in loading. The loading in the first part of the study is centrifugal, representing the cruise speed of the space shuttle.

A factorial design,[13] rather than a parametric approach, was selected to perform this study. This design is considered economical when the influence of more than two variables is to be determined either experimentally or using simulation. A large number of unnecessary tests or simulation runs can be eliminated. This design helps to identify insignificant variables, thereby reducing the computation cost and complexity of the study. Another advantage is that this design provides the op-

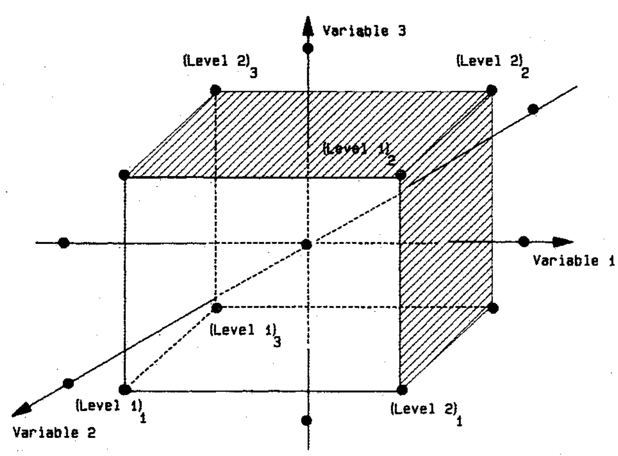

Fig. 3 Factorial design for three variables.

tion of widening the magnitude range of the variables by allowing the additional tests outside the preselected ranges. A factorial design for three variables is illustrated in Fig. 3. The limitation of the design is that the results, or the models developed using these results, are only applicable within the range covered by the design. In the parametric approach, one variable is varied at a time. The number of test runs needed to perform such studies using parametric approach is enormous.

For the present study, the number of variables was six. The variables for the geometry study were three means and three standard deviations of the perturbations along the three directions. The means were represented by μ_1, μ_2, and μ_3, and the standard deviations by σ_1, σ_2, and σ_3. For the material study, the six variables are the standard deviations of the perturbations property-matrix components. These standard deviations are represented by $\sigma_{c_{11}}$–$\sigma_{c_{66}}$. The variables in the combined study are the standard deviations of the dominant six variables out of the geometry and material-property studies. They are the standard deviations of the perturbations of coordinates represented by σ_1–σ_3 and the standard deviations of diagonal terms of the material-property matrix represented by $\sigma_{c_{11}}$, $\sigma_{c_{22}}$, and $\sigma_{c_{33}}$.

Model and Probability Distribution

Probabilistic models were developed for all response variables for each part of the study using regression analysis. Coefficients and their standard deviations for each of the study variables were estimated. Using standard deviations and t tests, it was determined whether a given variable was significant or not. After all the models were developed, F tests and χ^2-square tests were also used. These tests provided goodness-of-fit of the models and a measure of randomness of the residuals. Models were selected based on these tests. These models are of the form:

$$\hat{Y} = \alpha_0 + \alpha_1 V_1 + \alpha_2 V_2 + \alpha_3 V_3 + \cdots + \alpha_n V_n$$

where $\hat{Y}$ is a response variable, $\alpha_0,\cdots,\alpha_n$ are the coefficients, and $V_1,\cdots,V_n$ are the variables of the study. The residuals of the model were plotted and their autocorrelations were plotted before a model was finally selected or rejected. A sample plot of the residuals for one of the models is shown in Fig. 4. This plot indicates that the residuals are randomly distributed. Probability distributions for all of response variables were developed for selected simulated runs by replicating them a large number of times. Every replication used different random numbers for perturbations.

Results and Analysis

A methodology to perform probabilistic structural analysis has been developed. It can be used to quantify the influence of random variations or uncertainties in the structural variables on the performance of a given component. The methodology was developed using computer simulation of test runs. Simulation not only reduced high costs and complexity of experimentation, but it shortened the time of developing the results. The results reported in this study, as mentioned before, are all computer-simulated results. The magnitudes of the coefficients α_1, $\alpha_2,\cdots,\alpha_n$ for geometric variables, material-property variables, and cross terms (interaction terms) are listed in Tables 1, 2, and 3, respectively.

The significance test (t test) of the coefficients in Table 1 indicated that only variance of the perturbations along the thickness is significant. It is because variation in thickness has an exponential effect on stiffness. The same conclusions were reported by Belytschko and Liu.[11] The perturbations along the other directions are insignificant and should be considered insignificant while setting the tolerances. The probable reason that these perturbations are insignificant is that they are relatively small in magnitude. Although F tests showed that all the models are good fits, they are not the desired models since they include insignificant variables. A model using the significant variable alone was also developed (not shown here). This model predicted estimates close to the actual values and was found to be a good fit.

Probabilistic distributions of all of the response variables were obtained based on 120–500 simulated replications. Figure 5 shows the probability distributions of the first natural frequency, and Fig. 6 shows the probability distributions of root stress. These distributions provide estimates of two items: 1) a range of variation of a variable for any given random variation in the geometric variables, and 2) probability of occurrence of a response variable. This information is valuable to the designer in the adjustment of the design tolerance limits. If the range does not include any critical value, the tolerances can be relaxed. This will reduce manufacturing cost. These estimates can be further improved if the number of replication is increased.

Significance tests were performed on the coefficients of the material-property model (Table 2). These tests indicated that

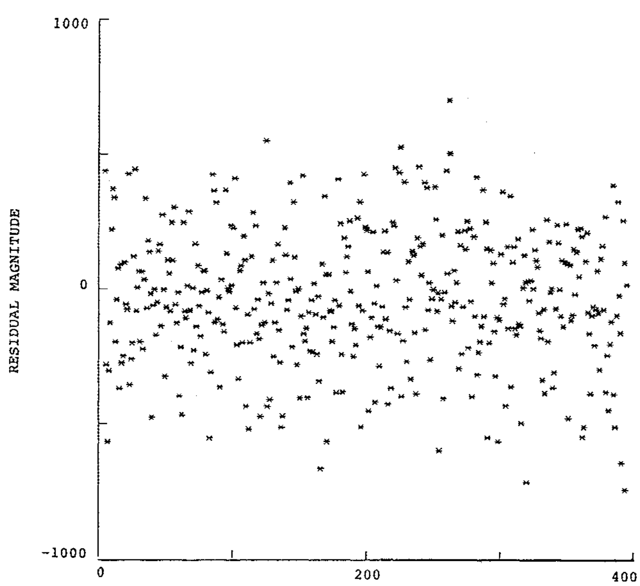

Fig. 4 Random distribution of residuals.

Table 1 Probabilistic models[a]—geometric perturbations

Model: Dependent variable = Constant + Coeff.* μ_1 + Coeff.* μ_2 + Coeff.* μ_3 + Coeff.* σ_1 + Coeff.* σ_2 + Coeff.* σ_3

		Coefficients of					
Dependent variable	Constant	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3
First frequency	6105.0	−832	−2915	−687	−8362	1897	−9348
Second frequency	9475.1	−1374	−2961	−408	−11398	−3869	−9095
Third frequency	15792.0	−9434	27592	−7944	−17663	−4899	−44147
Root stress	63323.0	49707	103320	4177	88960	−48191	283960
Tip displacement	0.00196	0.0119	0.1287	0.0206	−0.0132	−0.0419	0.0067

[a]F tests indicated that all models are good fits.

Table 2 Probabilistic models[a]—material-properties perturbations (different blade properties)

Model: Dependent variable = $\bar{Y}$ = Constant + Coeff.$_1$* C_{11} + Coeff.$_2$* C_{12} + Coeff.$_3$* C_{13} + Coeff.$_4$* C_{22} + Coeff.$_5$* C_{23} + Coeff.$_6$* C_{33}

		Coefficients of					
Dependent variable $\bar{Y}$	Constant	C_{11}	C_{12}	C_{13}	C_{22}	C_{23}	C_{33}
First frequency	4769.7	−239.1	−64.1	−55.1	−69.0	63.2	47.1
Second frequency	5501.9	−342.6	−25.5	−96.7	−250.7	110.4	52.2
Third frequency	9653.0	−788.6	−0.8	−61.5	−909.6	185.7	−63.6
Root stress	97673.0	498.0	−4001.0	−1336.0	−9962.0	−12154.0	16701.0
Tip displacement 1	0.0121	0.0031	0.0004	0.0001	0.0011	0.0005	0.00133
Tip displacement 2	0.0113	0.0029	−0.0006	0.0008	0.0008	0.0004	0.00133
Tip displacement 3	0.0012	0.0023	−0.0004	0.0001	0.0003	0.0007	0.0010

[a]F tests indicated that the models are not good fits.

Table 3 Probabilistic models[a,b]—geometric and material perturbations together

Model: Dependent variable Y = Constant + Coeff.$_1$ σ_1 + ⋯ + Coeff.$_6$ σ_{33} + ⋯ + Coeff.$_{21}$ σ_{33} $\sigma_{C_{33}}$

		Coefficients of					
Dependent variable Y	Constant	σ_1	σ_2	σ_3	$\sigma_{C_{11}}$	$\sigma_{C_{22}}$	$\sigma_{C_{33}}$
First frequency	6185.67	−336.2	320.6	−1588.1	−60.82	−15.37	−44.90
Second frequency	8901.47	146.00	−299.00	−524.00	−40.10	−68.4	−70.30
Third frequency	15267.0	17.40	−2307.00	−12324.00	−79.70	−294.00	−106.00
Root stress	85450.0	−25410.0	−39280.0	−80.0	−2080.0	−3260.0	−1920.0
Tip displacement 1	−1.66	−0.97	3.29	−3.05	−0.05	0.14	−0.11
Tip displacement 2	2.04	1.46	1.99	1.20	−0.12	−0.12	0.09
Tip displacement 3	−3.14	−2.31	−2.20	0.70	−0.16	0.12	−0.19

[a]Coefficients of all cross terms (not shown) and material property were insignificant. [b]Tip displacements coefficients to be divided by 1000.

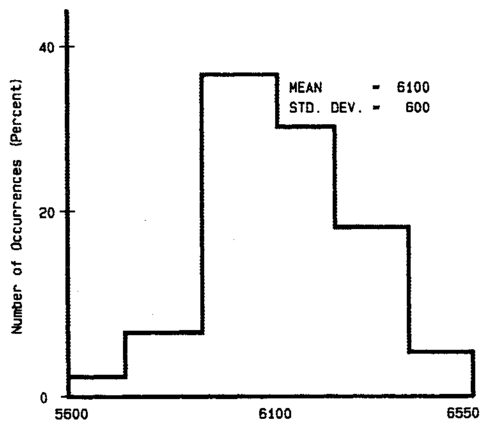

Fig. 5 Estimated probability distribution of first natural frequency —geometry perturbation.

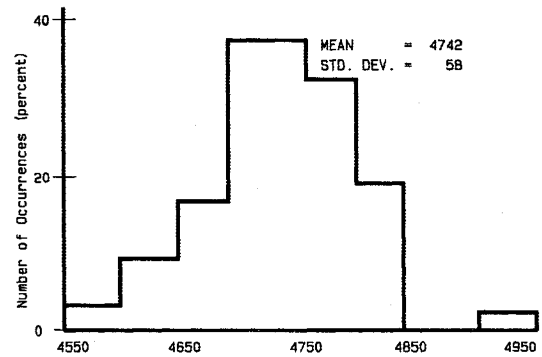

Fig. 7 Estimated probability distribution of first natural frequency —materials-properties perturbation.

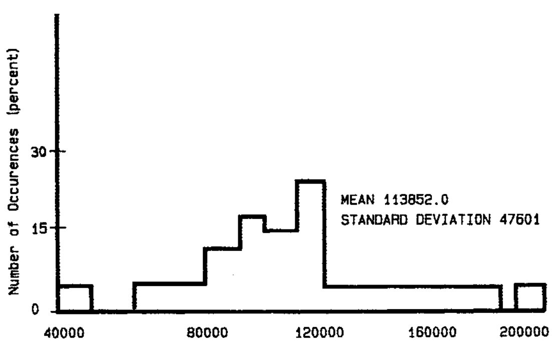

Fig. 6 Estimated probability distribution of root stress—geometry perturbation.

all of the coefficients are statistically insignificant. This means that variation in material properties within 10% did not have any significant influence on the response variables. Belytschko and Liu[11] reported the same findings. The variations of the response variables are truly random. In other words, these variations can be modeled as white noise. Therefore, controlling the material-property variations would not necessarily give desirable response. The distribution of all of response variables corresponding to the variation in material properties were obtained. A sample plot of the variation in the first natural frequency is shown in Fig. 7. This distribution provides estimates of the range and variations in the first natural frequency. The three histograms (distributions) included in this paper are a part of a large number of histograms developed in this study.

Significance tests were also performed on the coefficients (Table 3) of models developed to estimate the effects of interactions. These tests indicated that all of the interaction effects are statistically insignificant. This means that response to simultaneous variation in geometry will not be affected by simultaneous variation in material properties.

The statistical analysis was used to eliminate any insignificant variables from the study. Since this part of the main study is continuing, these interim conclusions help to reduce computer costs. Work is continuing in developing higher-order models; these models include estimating variances of the response variables. Some interesting results have been obtained. These will be published in the near future, along with the results from other parts.

Future Work

This study is being continued. The future work includes determining the influence of correlated random variation in geometry, material properties, temperature, pressure, and other loadings. The other loadings include random pulse loading, thermal loading, and random loading in elastic and inelastic ranges.

Conclusions

1) A methodology has been developed to quantify the influence of random variations or uncertainties in the geometry, material properties, and loadings on the response variables. This methodology provides answers which may be used as a guideline to set the tolerance limits for design purposes.

2) Within the limitations of this pilot study, it was found that random variations or uncertainties in geometry have statistically significant influence on the response variables. Therefore, tolerance limits should give due consideration to geometric variations along thickness.

3) Within the limitations of this pilot study, random variations or uncertainties in material properties have statistically insignificant effects on the response variables. Therefore, tolerance on material-property variations need not be precise.

4) The methodology can be used for identifying the insignificant variables.

5) The methodology is generic in nature and can be extended to the analysis of the other structural components.

Acknowledgment

The authors are thankful to NASA for the use of their computer facility and to Dr. Michael Shiao for his invaluable help in completing this part of the study.

References

[1]Huang, T. C. and Nagpal, V. K., "Probabilistic Factors, Experimental Design, and Statistical and Variance Analyses for Fatigue under Random Vibrations," *Random Vibrations,* edited by T. C. Huang and P. D. Spanos, AMD, Vol. 65, American Society of Mechanical Engineers, New York, Dec. 1984, pp. 51–68.

[2]Dover, W. D., "Fatigue Crack Growth in Tubular Welded Connections," *Second International Conference on Behavior of Offshore Structures,* London, England, Boss '79 Paper 40, Aug. 1979.

[3]Yang, J. N., "Reliability Analysis of Aircraft Structures," *Proceedings of Symposium on Probabilistic Methods in Structural Engineering,* American Society of Civil Engineers, New York, Oct. 1981, pp. 102–120.

[4]Wirshing, P. H., "Fatigue Reliability Analysis in Offshore Structures," *Proceedings of Symposium on Probabilistic Methods in Structural Engineering,* American Society of Civil Engineers, New York, Oct. 1981, pp. 295–307.

[5]Kawamoto, J., Shyam Sunder, S., and Connor, J. J., "An Assessment of Uncertainties in Fatigue Analyses of Steel Jacket Offshore Platform," *Applied Ocean Research,* Vol. 4, Jan. 1982, pp. 9–16.

[6]Miner, M. A., "Cumulative Damage in Fatigue," *Journal of Applied Mechanics,* Vol. 12, No. 3, 1945, pp. A159–163.

[7]Kurth, R., "Composite Load Spectra for Select Space Propulsion Structural Components, Structural Integrity, and Durability of Reusable Space Propulsion Systems," NASA CP-2381, 1985.

[8]Shinozuka, M. and Tan, R., "Probabilistic Load Combinations and Crossing Rate," *Proceedings of Symposium on Probabilistic Methods in Structural Engineering,* American Society of Civil Engineers, New York, Oct. 1981, pp. 229–250.

[9]Lin, Y. K., "Random Vibrations of Civil Engineering Structures," *Proceedings of the Symposium on Probabilistic Methods in Structural Engineering,* American Society of Civil Engineers, New York, Oct. 1981, pp. 210–228.

[10]DasGupta, G., "Literature Review on Probabilistic Structural Analysis and Stochastic Finite-Element Methods," NASA NAS3-24389, March 1986.

[11]Belytschko, T. and Liu, W. K., "Probabilistic Finite Elements: Variational Theory, Structural Integrity, and Durability of Reusable Space Propulsion Systems," NASA CP-2381, 1985.

[12]Chamis, C. C., "Probabilistic Structural Analysis Methods for Space Propulsion System Component," NASA TM-88861, 1987.

[13]Box, G. E. P., Hunter, W. G., and Hunter, J. S., *An Introduction to Design, Data Analysis, and Model Building,* Wiley, New York, 1979, Chaps. 9, 10, and 11.